Critical total strain-based Phase-Field Damage Model implementation for ductile fatigue

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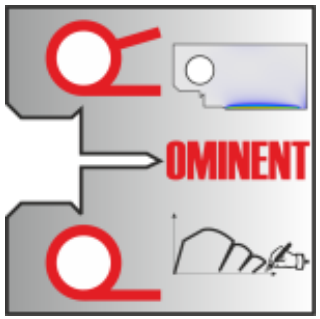
Outline

1. Introduction
2. Phase-field damage model
3. Fatigue degradation function
4. Example
5. Conclusions



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"Prediction of damage evolution in engineering structures - PROMINENT"

- development of FEM software PAK-DAM for the simulation of damage initiation, evolution and propagation to the fracture,
- experimental investigation of the materials (steel, aluminum, concrete),
- testing and verification of the software functionality and accuracy,
- budget cca 284.000e in period 2024-2026
- https://prominent.kg.ac.rs/

- Low cycle fatigue (LCF) is repeated loading-unloading of material that exhibits plastic strains.
- The loading amplitude is above the yield strength.
- Von Mises plasticity and Phase-field Damage model can be used to create multifield finite elements to compute damage as additional degree of freedom.
- In this work, we presented the simulation results of damage in metals (Von Mises plasticity model) by PFDM implementation of work-densities-based criterion with threshold [Miehe et al.][1].
- The fatigue degradation function [Simoes et al.][2] is implemented for simulating accumulation of damage in the material.

[1] Miehe C., Hofacker M., Schanzel L.-M., Aldakheel F. (2015) Phase field modeling of fracture in multi-physics problems. Part II. Coupled brittle-to-ductile failure criteria and crack propagation in thermo-elastic–plastic solids, CMAME, 294, 486-522

[2] Simoes, M, Braithwaite, C, Makaya, A, Martínez-Pañeda, E. (2022) Modelling fatigue crack growth in shape memory alloys. Fatigue Fract Eng Mater Struct, 45(4):1243-1257

Phase-field damage model

The formulation for cracks in one-dimensional solids, and the extension of the regularized crack functional to multi-dimensional problems, the crack surface is

$$S(d) = \int_V \gamma(d, \nabla d)\, dV \tag{1}$$

a crack surface density function γ per unit volume is defined as

$$\gamma(d, \nabla d) = \frac{d^2}{2l_c} + \frac{l_c}{2}|\nabla d|^2 \tag{2}$$

where ∇ is the gradient operator.

Phase-field damage model

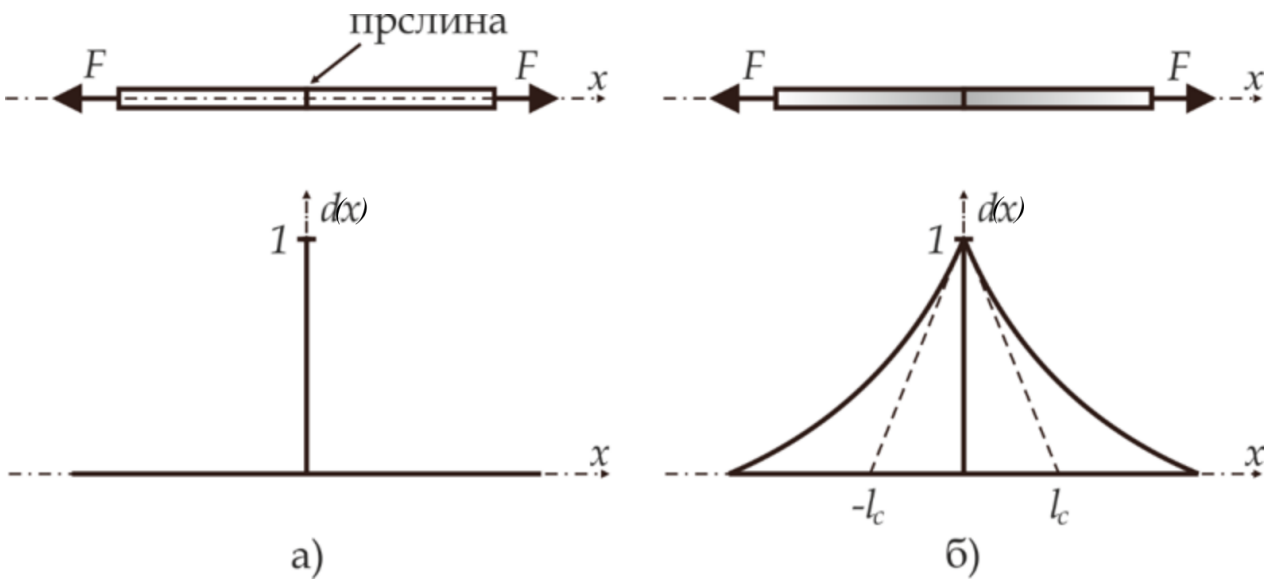


Figure: Crack topology a) discrete and b) diffusive

Phase-field damage variable:

$$d(x) = \begin{cases} 1 & x = 0 \\ 0 & x \neq 0 \end{cases} \qquad d(x) = e^{-\frac{|x|}{l_c}} \tag{3}$$

l_c is the characteristic length (crack width).

The internal potential energy density ψ is considered as the sum of elastic $\psi^E(\boldsymbol{\varepsilon}_E, d)$, plastic $\psi^P(\bar{\varepsilon}_P, d)$, and fracture contribution $\psi^{frac}(d, \nabla d)$ energy density:

$$\psi = \psi^E(\boldsymbol{\varepsilon}_E, d) + \psi^P(\bar{\varepsilon}_P, d) + \psi^{frac}(d, \nabla d) \tag{4}$$

The total internal potential energy W_{int} functional is defined according to Miehe et al.[3] as:

$$W_{int} = \int_V \psi dV = \int_V \{g(d)\left(\psi^E(\boldsymbol{\varepsilon}_E, d) + \psi^P(\bar{\varepsilon}_P, d)\right) + G_V\left[d + \frac{l_c^2}{2}|\nabla d|^2\right]\} dV$$

where a critical fracture energy release rate per unit volume is $G_V = \frac{G_c}{l_c}$.
Degradation function is defined:

$$g(d) = (1-d)^2 \tag{5}$$

[3] Miehe C., Hofacker M., Schanzel L.-M., Aldakheel F. (2015) Phase field modeling of fracture in multi-physics problems. Part II. Coupled brittle-to-ductile failure criteria and crack propagation in thermo-elastic–plastic solids, CMAME, 294, 486-522

The equilibrium of the internal and external potential energy

$$\delta W_{int} = \delta W_{ext} \tag{6}$$

leads to the equilibrium equation:

$$Div\left[\boldsymbol{\sigma}\right] + \mathbf{b} = 0 \tag{7}$$

plasticity yield condition law:

$$\bar{\sigma}_{eq} - \sigma_{yv} - \left(\sigma_{y0,\infty} - \sigma_{yv}\right)\left(1 - e^{-n\bar{\varepsilon}_P}\right) - H\bar{\varepsilon}_P = 0 \tag{8}$$

and the phase field damage evolution law according to Miehe et al.[4] :

$$G_V\left[d - l_c^2\nabla^2 d\right] + g'\left(d\right) max\left(\psi^E + \psi^P - \frac{G_v}{2}\right) = 0 \tag{9}$$

[4] Miehe C., Hofacker M., Schanzel L.-M., Aldakheel F. (2015) Phase field modeling of fracture in multi-physics problems. Part II. Coupled brittle-to-ductile failure criteria and crack propagation in thermo-elastic–plastic solids, CMAME, 294, 486-522

Critical total strain vs fracture energy

$$G_v = E\left[\bar{\varepsilon}_{cr}^2 - \left(\bar{\varepsilon}_{cr}^2 - \frac{\sigma_y}{E}\right)^2\right] + (\sigma_{y1} - \sigma_y)\left(\bar{\varepsilon}_{cr} - \frac{\sigma_y}{E}\right) \tag{10}$$

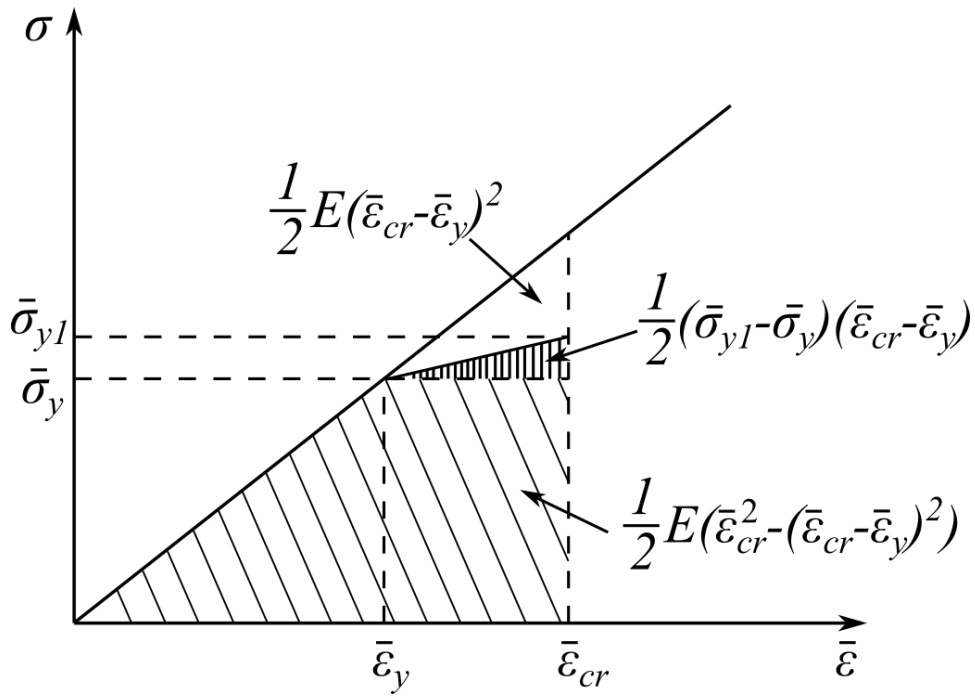


Figure: Fracture energy value and critical total strain

Fatigue degradation function

The field equation corresponding to phase field variable is given as[5]:

$$f\left(\bar{\alpha}(t)\right) G_V(\xi)\left[d - l_c^2 \nabla^2 d\right] + g'(d)\,\psi = 0 \tag{11}$$

where $\bar{\alpha}(t)$ is cumulation of any scalar quantity which can describe the fatigue history experienced by the material. For a pseudo-time τ, the cumulative history variable can be defined as:

$$\bar{\alpha}(t) = \int_0^t H(\alpha\dot{\alpha})|\dot{\alpha}|d\tau \tag{12}$$

where $H(\alpha\dot{\alpha})$ is the Heaviside step function, defined as $H(\alpha\dot{\alpha}) = 1$ if $\alpha\dot{\alpha} \geq 0$ (loading) and $H(\alpha\dot{\alpha}) = 0$ otherwise (unloading).

[5] Simoes, M, Braithwaite, C, Makaya, A, Martínez-Pañeda, E. Modelling fatigue crack growth in shape memory alloys. Fatigue Fract Eng Mater Struct. 2022; 45(4): 1243-1257

- $\bar{\alpha}$ only grows during loading.
- $\alpha = g(d)\psi$ is the fatigue history variable represents the loading conditions in the solid
- $f(\bar{\alpha}(t))$ is the fatigue degradation function characterizes the sensitivity of the fracture energy to the number of cycles.

$$f(\bar{\alpha}(t)) = \begin{cases} 1 & \text{if} \quad \bar{\alpha} \leq \alpha_T \\ \left(\frac{2\alpha_T}{\bar{\alpha}(t)+\alpha_T}\right)^2 & \text{if} \quad \bar{\alpha} > \alpha_T \end{cases} \tag{13}$$

- α_T represents a threshold value, below which the fracture energy remains unaffected:

$$\alpha_T = \frac{G_V}{2} \tag{14}$$

Uniaxial loading of unit cube

The unit cube is loaded by prescribed displacement on one side, while the other side is restrained.

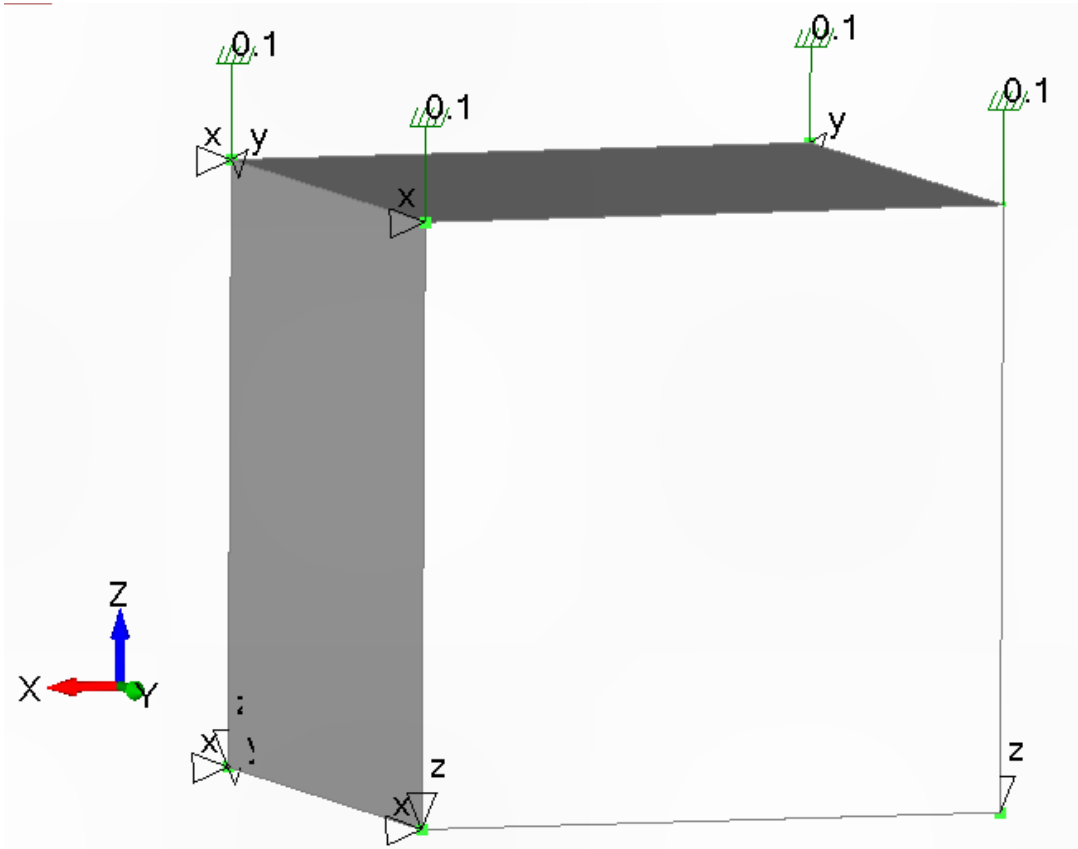


Figure: Boundary and loading conditions of uniaxial loading example

Table: Elasto-plasticity material parameters

E	ν	n
199000 MPa	0.29	18 MPa
σ_{y0}	$\sigma_{y\infty}$	H
345 MPa	635 MPa	9 MPa [-]

Table: Phase-Field damage model parameters

G_v	$\overline{e}^{crit}$	l_c
82.2 MPa	0.012	0.1 mm

Three loading-unloading cycles are performed up to strain: the first cycle = 2%, the second cycle = 3%, the third cycle 4%.

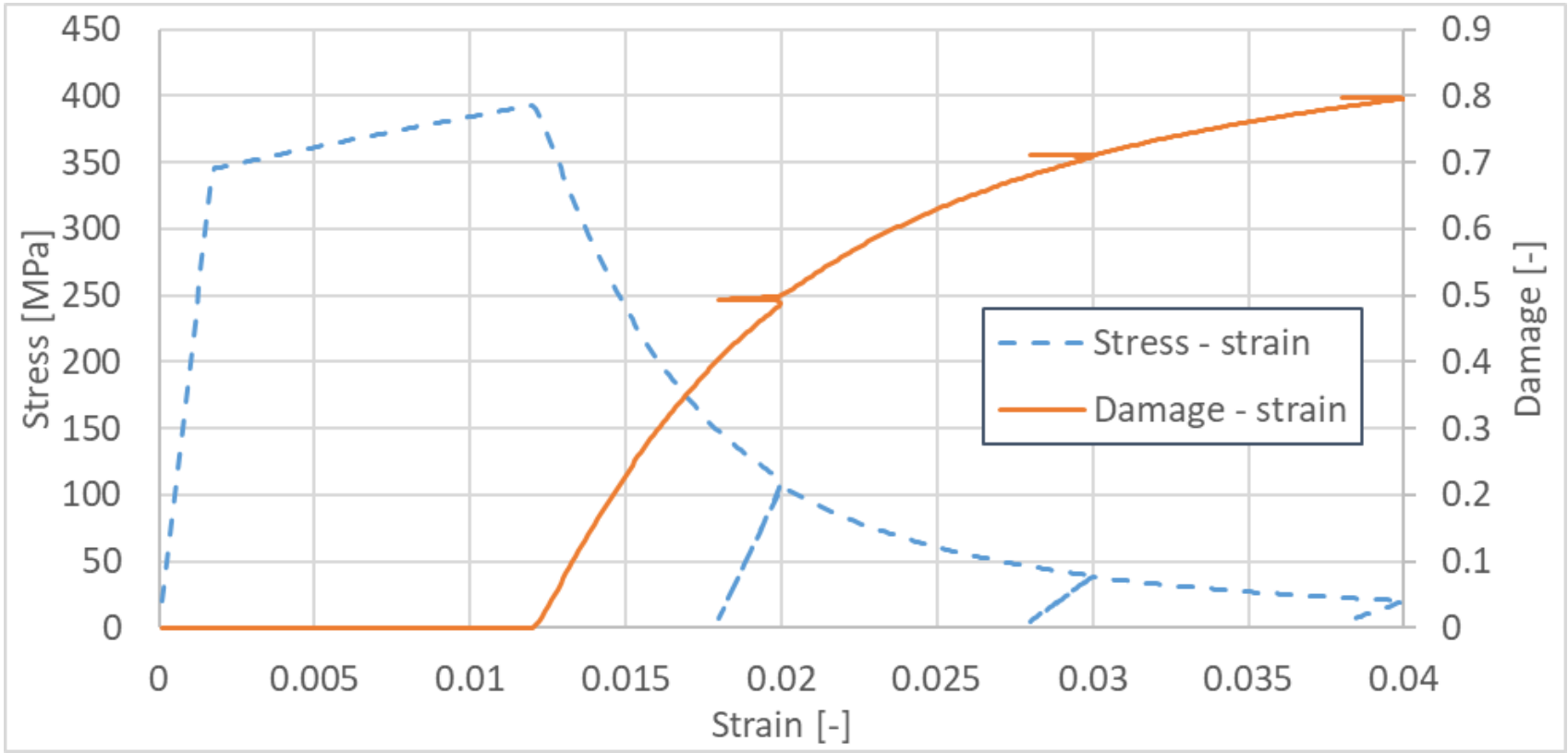
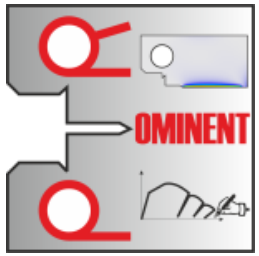


Figure: Stress-strain and damage-strain response

Conclusions

- PFDM is often implemented into FEM for computation of damage and fracture in structures
- The fatigue degradation function opens the possibility to simulate accumulation of damage and fatigue
- Von Mises plasticity constitutive model with Simo hardening function with PFDM and fatigue degradation function is used to show influence of cyclic loading to ductile behavior of metals
- Simple unit cube model is used to show phenomenology and implementation.
- The behavior is shown by stress-strain and damage-strain relationships.



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- Dunic, V., Zivkovic, J., Milovanovic, V., Pavlovic, A., Radovanovic, A., Zivkovic, M. (2021) Two-Intervals Hardening Function in a Phase-Field Damage Model for the Simulation of Aluminum Alloy Ductile Behavior. Metals, vol 11(11), pp. 1685
- V.Dunic, R.Matsui, K.Takeda, M.Zivkovic, Phase-field damage simulation of subloop loading in TiNi SMA, (**2024**) International Journal of Damage Mechanics, Vol. 33(8), pp. 577-604 - SAKURA Exchange Program 2023

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